Análisis Multivariado

Producto académico 03

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# Ejercicios AFE, Cluster y análisis de correspondencia múltiple

## CASO 1: Postulantes (6 puntos)

Se tienen datos de las notas de alumnos postulantes a un colegio de alto rendimiento, se desea agrupar las notas de los cursos y ver que grupos podrían haber de cursos.

Las notas de los siguientes cursos son Razonamiento verbal, Razonamiento matemático, Matemáticas, Psicología y filosofía, Física, Lógica, Biología, Historia y Química.

Archivo a utilizar **postulantes.sav**

library(foreign)  
postulantes <- read.spss(here("9 Analisis Multivariado/Trabajo 3/Postulantes.sav"),  
 to.data.frame = TRUE,  
 use.value.labels = TRUE)

1. Realizar análisis factorial exploratorio
2. Decida cuantos factores retener explique el por qué.
3. Decida el método de rotación y explique el por qué.
4. Explicar los resultados y de sus conclusiones del ejercicio.

### Solución

1. **Realizar análisis factorial exploratorio**

Veamos un resumen de los datos

summary(postulantes)

## ID RV RM MAT   
## Length:541 Min. : 0.50 Min. :-1.08 Min. :-1.250   
## Class :character 1st Qu.: 8.75 1st Qu.: 7.85 1st Qu.: 6.660   
## Mode :character Median :10.50 Median :10.71 Median : 9.790   
## Mean :10.27 Mean :10.14 Mean : 9.301   
## 3rd Qu.:12.50 3rd Qu.:12.85 3rd Qu.:12.290   
## Max. :16.75 Max. :18.57 Max. :18.950   
## PSI FIS LOG BIO   
## Min. : 4.18 Min. :-2.150 Min. :-0.90 Min. :-3.930   
## 1st Qu.: 7.53 1st Qu.: 5.350 1st Qu.: 7.69 1st Qu.: 6.420   
## Median :10.31 Median : 8.920 Median :10.17 Median :10.350   
## Mean :10.35 Mean : 8.351 Mean :10.07 Mean : 9.703   
## 3rd Qu.:13.08 3rd Qu.:11.780 3rd Qu.:12.76 3rd Qu.:13.570   
## Max. :16.53 Max. :20.000 Max. :19.37 Max. :20.000   
## HIS QUI   
## Min. :-1.36 Min. :-2.50   
## 1st Qu.: 8.72 1st Qu.: 8.92   
## Median :10.97 Median :12.85   
## Mean :10.65 Mean :11.57   
## 3rd Qu.:13.10 3rd Qu.:15.00   
## Max. :19.42 Max. :20.00

No se tienen datos vacíos por lo cual no es necesario realizar una técnica de imputación, por lo que empezemos omitiendo la primera columna que representa a los códigos de los postulantes

postulantes <- postulantes[,-1]

#### Prueba de esfericidad de Bartlet

library(psych)  
cortest.bartlett(cor(postulantes), n=nrow(postulantes))

## $chisq  
## [1] 1968.098  
##   
## $p.value  
## [1] 0  
##   
## $df  
## [1] 36

Se nos muestra un p valor menor a 0.05, justifica el uso de reducción de datos.

#### Indicador Kaiser-Meyer-Olkinn KMO y MSA

KMO(postulantes)

## Kaiser-Meyer-Olkin factor adequacy  
## Call: KMO(r = postulantes)  
## Overall MSA = 0.78  
## MSA for each item =   
## RV RM MAT PSI FIS LOG BIO HIS QUI   
## 0.61 0.86 0.81 0.47 0.85 0.61 0.86 0.69 0.86

Los valores son menores a 0.5 a excepción de PSI (Psicología) por lo que se puede extraer para que se considere aceptable la aplicación del análisis factorial al conjunto de datos

data\_AFE <- postulantes[,-4]

Veamos nuevamente el test de bartlet y KMO

cortest.bartlett(cor(data\_AFE), n=nrow(data\_AFE))

## $chisq  
## [1] 1866.656  
##   
## $p.value  
## [1] 0  
##   
## $df  
## [1] 28

Significativo, ahora veamos el KMO

KMO(data\_AFE)

## Kaiser-Meyer-Olkin factor adequacy  
## Call: KMO(r = data\_AFE)  
## Overall MSA = 0.79  
## MSA for each item =   
## RV RM MAT FIS LOG BIO HIS QUI   
## 0.61 0.86 0.81 0.85 0.84 0.86 0.69 0.86

Ahora si es justificable el uso de un análisis factorial exploratorio

1. **Decida cuantos factores retener explique el por qué.**

Empezemos realizando la primera seleccionando tomando en cuenta todos los factores

facto=principal(r=data\_AFE,nfactors=8,rotate="none")  
facto

## Principal Components Analysis  
## Call: principal(r = data\_AFE, nfactors = 8, rotate = "none")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 h2 u2 com  
## RV 0.52 0.76 -0.16 -0.18 -0.03 0.03 0.01 0.32 1 0.0e+00 2.4  
## RM 0.66 -0.28 0.26 -0.52 0.36 0.06 -0.11 -0.01 1 1.0e-15 3.4  
## MAT 0.78 -0.35 0.15 -0.17 -0.27 0.00 0.38 0.01 1 1.8e-15 2.4  
## FIS 0.80 -0.29 0.02 0.12 -0.32 0.24 -0.31 0.03 1 1.8e-15 2.2  
## LOG 0.25 0.42 0.83 0.28 0.02 -0.03 0.00 -0.02 1 2.2e-16 2.0  
## BIO 0.74 -0.04 -0.24 0.46 0.33 0.23 0.14 -0.01 1 1.0e-15 2.8  
## HIS 0.66 0.61 -0.22 -0.14 -0.08 -0.04 -0.02 -0.34 1 5.6e-16 2.9  
## QUI 0.81 -0.22 -0.13 0.19 0.05 -0.48 -0.10 0.06 1 1.4e-15 2.1  
##   
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8  
## SS loadings 3.66 1.46 0.92 0.69 0.42 0.35 0.28 0.22  
## Proportion Var 0.46 0.18 0.11 0.09 0.05 0.04 0.03 0.03  
## Cumulative Var 0.46 0.64 0.75 0.84 0.89 0.94 0.97 1.00  
## Proportion Explained 0.46 0.18 0.11 0.09 0.05 0.04 0.03 0.03  
## Cumulative Proportion 0.46 0.64 0.75 0.84 0.89 0.94 0.97 1.00  
##   
## Mean item complexity = 2.5  
## Test of the hypothesis that 8 components are sufficient.  
##   
## The root mean square of the residuals (RMSR) is 0   
## with the empirical chi square 0 with prob < NA   
##   
## Fit based upon off diagonal values = 1

facto$loadings

##   
## Loadings:  
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8   
## RV 0.515 0.757 -0.160 -0.183 0.316  
## RM 0.664 -0.278 0.258 -0.517 0.363 -0.109   
## MAT 0.782 -0.350 0.146 -0.171 -0.265 0.380   
## FIS 0.804 -0.290 0.123 -0.321 0.235 -0.307   
## LOG 0.250 0.418 0.826 0.280   
## BIO 0.741 -0.242 0.458 0.327 0.234 0.141   
## HIS 0.659 0.612 -0.217 -0.142 -0.341  
## QUI 0.807 -0.221 -0.127 0.194 -0.480   
##   
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8  
## SS loadings 3.662 1.458 0.918 0.691 0.423 0.349 0.279 0.221  
## Proportion Var 0.458 0.182 0.115 0.086 0.053 0.044 0.035 0.028  
## Cumulative Var 0.458 0.640 0.755 0.841 0.894 0.937 0.972 1.000

Por la mayor explicación de la varianza, se recomienda usar 3 o 4 factores. Decidamos el uso de 4 factores

1. **Decida el método de rotación y explique el por qué.**

Teniendo en cuenta que se tendrán 4 factores veamos las cargas factoriales

facto=principal(r=data\_AFE,nfactors=4,rotate="none")  
facto

## Principal Components Analysis  
## Call: principal(r = data\_AFE, nfactors = 4, rotate = "none")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## PC1 PC2 PC3 PC4 h2 u2 com  
## RV 0.52 0.76 -0.16 -0.18 0.90 0.1021 2.0  
## RM 0.66 -0.28 0.26 -0.52 0.85 0.1483 2.6  
## MAT 0.78 -0.35 0.15 -0.17 0.79 0.2146 1.6  
## FIS 0.80 -0.29 0.02 0.12 0.75 0.2534 1.3  
## LOG 0.25 0.42 0.83 0.28 1.00 0.0019 2.0  
## BIO 0.74 -0.04 -0.24 0.46 0.82 0.1816 1.9  
## HIS 0.66 0.61 -0.22 -0.14 0.88 0.1239 2.3  
## QUI 0.81 -0.22 -0.13 0.19 0.75 0.2461 1.3  
##   
## PC1 PC2 PC3 PC4  
## SS loadings 3.66 1.46 0.92 0.69  
## Proportion Var 0.46 0.18 0.11 0.09  
## Cumulative Var 0.46 0.64 0.75 0.84  
## Proportion Explained 0.54 0.22 0.14 0.10  
## Cumulative Proportion 0.54 0.76 0.90 1.00  
##   
## Mean item complexity = 1.9  
## Test of the hypothesis that 4 components are sufficient.  
##   
## The root mean square of the residuals (RMSR) is 0.06   
## with the empirical chi square 95.01 with prob < 2.3e-21   
##   
## Fit based upon off diagonal values = 0.98

facto$loadings

##   
## Loadings:  
## PC1 PC2 PC3 PC4   
## RV 0.515 0.757 -0.160 -0.183  
## RM 0.664 -0.278 0.258 -0.517  
## MAT 0.782 -0.350 0.146 -0.171  
## FIS 0.804 -0.290 0.123  
## LOG 0.250 0.418 0.826 0.280  
## BIO 0.741 -0.242 0.458  
## HIS 0.659 0.612 -0.217 -0.142  
## QUI 0.807 -0.221 -0.127 0.194  
##   
## PC1 PC2 PC3 PC4  
## SS loadings 3.662 1.458 0.918 0.691  
## Proportion Var 0.458 0.182 0.115 0.086  
## Cumulative Var 0.458 0.640 0.755 0.841

Vemos que las cargas aportan a cada factor siendo posible su distinción entre factores teniendo los siguientes resultados

* PC1: **RM (confuso)**, MAT, FIS, BIO **HIS (confuso)** y QUI
* PC2: RV, **HIS (confuso)**
* PC3: LOG
* PC4: **RM (confuso)** y BIO

Siendo casos en donde no se observan las diferencias de una variable hacia el factor, por lo que es necesario explicar la máxima varianza, veamos con varimax

facto=principal(r=data\_AFE,nfactors=4,rotate="varimax")  
facto

## Principal Components Analysis  
## Call: principal(r = data\_AFE, nfactors = 4, rotate = "varimax")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## RC1 RC2 RC4 RC3 h2 u2 com  
## RV 0.09 0.93 0.05 0.13 0.90 0.1021 1.1  
## RM 0.15 0.13 0.90 0.04 0.85 0.1483 1.1  
## MAT 0.49 0.06 0.73 0.05 0.79 0.2146 1.8  
## FIS 0.71 0.08 0.49 0.07 0.75 0.2534 1.8  
## LOG 0.03 0.14 0.06 0.99 1.00 0.0019 1.1  
## BIO 0.87 0.24 0.05 0.03 0.82 0.1816 1.2  
## HIS 0.27 0.88 0.15 0.06 0.88 0.1239 1.2  
## QUI 0.77 0.16 0.37 -0.01 0.75 0.2461 1.5  
##   
## RC1 RC2 RC4 RC3  
## SS loadings 2.19 1.78 1.76 1.00  
## Proportion Var 0.27 0.22 0.22 0.13  
## Cumulative Var 0.27 0.50 0.72 0.84  
## Proportion Explained 0.33 0.26 0.26 0.15  
## Cumulative Proportion 0.33 0.59 0.85 1.00  
##   
## Mean item complexity = 1.3  
## Test of the hypothesis that 4 components are sufficient.  
##   
## The root mean square of the residuals (RMSR) is 0.06   
## with the empirical chi square 95.01 with prob < 2.3e-21   
##   
## Fit based upon off diagonal values = 0.98

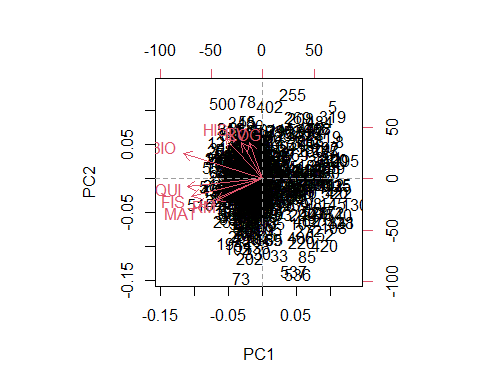
facto$loadings

##   
## Loadings:  
## RC1 RC2 RC4 RC3   
## RV 0.933 0.125  
## RM 0.146 0.127 0.901   
## MAT 0.491 0.734   
## FIS 0.705 0.489   
## LOG 0.142 0.987  
## BIO 0.870 0.239   
## HIS 0.267 0.883 0.145   
## QUI 0.770 0.156 0.369   
##   
## RC1 RC2 RC4 RC3  
## SS loadings 2.191 1.779 1.756 1.003  
## Proportion Var 0.274 0.222 0.220 0.125  
## Cumulative Var 0.274 0.496 0.716 0.841

Veamos ahora los factores, asimismo como su importancia

* RC1: FIS, BIO y QUI
* RC2: RV e HIS
* RC4: RM, MAT
* RC3: LOG

biplot(prcomp(data\_AFE, scale = FALSE))  
abline(h = 0, v = 0, lty = 2, col = 8)



Como se observa hay mejor distinción de las variables con respecto a sus factores asimismo podemos explicar cada factor de la siguiente manera

* **RC1 (Ciencias naturales):** Conformada por Física, biología y química
* **RC2 (Comprensión información):** Conformada por razonamiento verbal e historia
* **RC4 (Ciencias matemáticas):** Conformada por razonamiento matemático y matemáticas
* **RC4 (Lógica):** Conformada por lógica

Concluyendo que la rotación varimax permite una mejor distinción entre factores a través de la explicación de su varianza máxima asimismo como los componentes creados tienen lógica con el contexto del ejercicio.

1. **Explicar los resultados y de sus conclusiones del ejercicio.**

Saquemos ahora los scores

scores <- as.matrix(data\_AFE) %\*% as.matrix(facto$loadings)  
scores <- data.frame(scores) ; scores

## RC1 RC2 RC4 RC3  
## 1 13.376612 26.616632 17.4169641 15.6887670  
## 2 34.545960 27.366521 23.7660459 15.2176690  
## 3 38.127554 23.167333 33.6622523 6.6451269  
## 4 51.435037 34.528372 42.7293245 14.0623244  
## 5 5.254995 24.611045 0.4274800 12.5820336  
## 6 23.483688 29.392048 22.5527796 15.2444195  
## 7 44.182112 29.939483 34.2706927 3.9555792  
## 8 1.580331 17.185895 2.9121549 14.5115115  
## 9 28.298864 25.704976 25.7634482 16.1169743  
## 10 42.231320 32.815369 29.4849396 5.3893371  
## 11 41.403656 25.787700 27.4514864 4.4281814  
## 12 11.818890 28.633332 7.6340880 7.5673792  
## 13 39.169357 31.820736 26.8272768 15.7927755  
## 14 45.101253 29.044654 30.9023942 16.8630148  
## 15 32.384606 31.817379 29.4833462 10.4967271  
## 16 42.866409 29.893208 31.2369665 7.7001236  
## 17 44.400553 30.814138 36.1198874 23.2761258  
## 18 49.958610 32.480019 39.9342484 6.0370644  
## 19 42.567005 19.597001 32.3700207 10.6472971  
## 20 35.304708 37.991081 27.6241939 13.9557084  
## 21 38.421169 29.224627 27.4209302 13.7477090  
## 22 34.860277 25.830289 30.3765724 11.3004796  
## 23 45.454818 33.264962 35.6199889 10.7385868  
## 24 17.800567 24.082659 18.3318084 1.9917228  
## 25 37.099720 35.268795 35.4943589 18.0651837  
## 26 50.532690 30.897721 41.0557550 14.0606513  
## 27 37.461159 29.389636 26.6555239 5.6564711  
## 28 25.475414 26.064329 20.2064226 11.7107044  
## 29 32.622556 27.601385 28.4181465 15.1424185  
## 30 40.809779 39.527506 35.0789412 19.4805479  
## 31 43.789011 35.372074 40.1755880 11.4879017  
## 32 16.213346 23.864087 21.2848170 14.4433914  
## 33 27.704216 12.135118 25.6324143 5.0415367  
## 34 41.266496 21.405072 31.6969121 10.0371183  
## 35 8.161107 25.136533 14.1704198 9.2197363  
## 36 40.712933 28.050522 26.1345698 15.8493646  
## 37 38.268695 32.664197 29.0414519 11.3305849  
## 38 34.252158 32.568184 37.8943645 16.9027570  
## 39 30.514940 26.430391 24.1449520 13.4340806  
## 40 38.535436 26.606386 31.9239399 11.6179432  
## 41 22.747951 31.770350 19.6212098 16.3033356  
## 42 45.944838 33.447229 29.8364744 17.0015631  
## 43 20.308438 25.958747 21.8897538 13.9949018  
## 44 18.158192 19.556881 21.3713091 10.3511787  
## 45 34.365554 31.535770 27.6531332 16.0066675  
## 46 39.527405 36.646090 32.5958801 19.0540833  
## 47 38.575212 35.198650 40.9889253 18.6413591  
## 48 30.975591 33.822059 30.1434649 18.3368514  
## 49 45.361963 31.804599 36.7549292 14.6033128  
## 50 23.164083 20.291949 19.3158444 9.5832235  
## 51 39.790667 30.104162 26.7897498 14.1980264  
## 52 33.131230 26.774394 34.4490168 12.8267937  
## 53 42.701050 29.299928 31.0647974 13.6598348  
## 54 42.220491 19.567675 35.7645796 7.4057117  
## 55 39.983765 36.654923 27.4987974 17.8038917  
## 56 44.418771 38.871697 31.6927067 19.5717687  
## 57 43.590584 36.017606 37.3893687 17.8575022  
## 58 48.871538 35.652636 37.1255757 17.7537148  
## 59 48.610702 29.783667 38.8373785 13.5033498  
## 60 46.007614 30.801200 36.1542443 14.9314950  
## 61 38.326700 33.681418 25.7029118 16.8904745  
## 62 41.431245 22.365910 30.3160032 9.4626907  
## 63 42.487493 28.772202 26.6870158 14.0803650  
## 64 46.801797 39.597560 35.8151589 19.5118335  
## 65 33.734958 30.425658 31.7535559 15.0411090  
## 66 32.181346 31.710460 28.3536514 15.4948030  
## 67 19.742368 18.813688 20.3816050 8.4877280  
## 68 52.232776 37.027238 39.7460195 18.5300034  
## 69 15.631888 27.268738 19.3650487 14.6002070  
## 70 42.547055 34.299788 33.1766155 17.1032030  
## 71 41.973535 33.208834 28.4185764 15.9715274  
## 72 27.129863 27.567241 20.8724986 13.1295257  
## 73 43.246086 16.554094 38.5923988 6.4722852  
## 74 40.629941 33.070794 30.0908259 15.7781915  
## 75 28.120416 27.070282 33.4897992 14.2880635  
## 76 5.668239 14.136642 16.2943662 7.0490875  
## 77 45.566687 34.133175 34.4405042 17.4837993  
## 78 40.092078 38.477809 25.4014657 18.3995653  
## 79 35.457427 27.808385 23.2781774 13.4900025  
## 80 29.972551 24.661730 25.6365718 11.3939031  
## 81 37.188610 24.526822 31.6357394 10.9179243  
## 82 42.878186 33.433017 33.9530958 15.3805741  
## 83 24.210284 30.704912 24.7814475 16.1339677  
## 84 32.364578 26.882623 29.0981389 13.4553380  
## 85 15.165655 9.522503 19.5761624 3.9035747  
## 86 36.298539 20.772498 30.7151118 9.7212219  
## 87 48.195356 30.414532 38.2053509 14.6625112  
## 88 46.331908 25.877343 32.8043381 11.9233322  
## 89 33.567381 24.781049 31.7124406 12.1227388  
## 90 9.440580 18.221322 11.1454181 8.9515666  
## 91 34.805576 32.806470 27.2041615 16.7543865  
## 92 39.794165 33.102680 32.4466230 15.3869638  
## 93 24.140311 20.919965 25.6879529 9.9908186  
## 94 36.845395 28.086264 33.9108626 13.7438477  
## 95 39.395363 30.642012 30.4495364 13.9891731  
## 96 35.772895 21.217056 26.8545426 9.3987580  
## 97 24.087123 23.744031 21.5081090 10.7777439  
## 98 14.307532 12.960632 10.9041563 5.6768890  
## 99 33.740943 18.953143 27.1584013 7.9084181  
## 100 30.459986 25.029919 25.7515678 12.5176402  
## 101 32.699798 33.304801 23.8327118 17.3395230  
## 102 45.055810 19.115528 36.5908986 7.1372894  
## 103 16.599938 15.220078 18.7989092 8.1889695  
## 104 40.229437 30.748954 34.1546477 14.6228881  
## 105 31.179607 17.737294 24.8035027 6.2773206  
## 106 23.943328 18.460098 14.8588669 12.1613546  
## 107 43.022786 31.573988 36.6343485 13.7329987  
## 108 5.172279 6.812855 8.0020519 4.3108007  
## 109 31.944976 26.217903 33.7899568 14.0938206  
## 110 28.035171 22.617370 28.1433347 12.9224433  
## 111 7.991326 12.238098 3.3274835 9.2517137  
## 112 46.373683 35.690441 37.4287493 17.6796849  
## 113 52.054437 37.814146 38.2914758 17.3796510  
## 114 38.146444 26.987275 37.0551107 12.3898705  
## 115 2.883631 10.311460 4.9295589 10.4571350  
## 116 42.490446 32.796743 34.5056951 15.4380377  
## 117 48.196970 32.187953 38.1691938 14.4342131  
## 118 43.446678 29.067010 31.1302182 14.4482794  
## 119 42.972334 30.031911 32.2267833 13.3758423  
## 120 16.455316 17.945047 13.8642408 11.3447719  
## 121 43.716706 28.977856 34.2726927 13.2731852  
## 122 41.343775 28.328887 32.9536811 11.3775651  
## 123 49.682435 39.993479 38.7684425 20.8436842  
## 124 35.657663 25.345884 32.0852850 13.3583366  
## 125 5.568225 8.075455 -0.2602340 6.4382973  
## 126 33.924918 28.445902 36.9499158 11.5242709  
## 127 48.681913 35.336045 41.4931704 15.7371790  
## 128 37.355524 26.438640 32.4157535 12.6445740  
## 129 34.879696 25.762650 30.2189070 12.3804216  
## 130 -3.523450 5.842727 0.4218768 7.8489903  
## 131 48.671838 33.547048 34.1520925 15.4047201  
## 132 36.022572 28.869477 28.7088481 15.7474131  
## 133 31.120117 28.836827 34.3355456 14.1806066  
## 134 39.793773 29.892263 26.0042127 15.4726018  
## 135 34.849928 28.846895 26.9950264 15.0399104  
## 136 35.812406 19.345002 33.5612469 6.2202023  
## 137 34.545816 25.641816 30.7324738 12.6097696  
## 138 32.829090 21.671981 29.6321437 9.0480311  
## 139 12.746768 17.139889 11.7641594 12.2179682  
## 140 48.558447 32.377612 33.0234992 15.6530680  
## 141 5.197910 12.297093 7.4664066 10.8029287  
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## 476 11.540746 27.614055 13.3651928 16.4847650  
## 477 27.943462 26.059587 29.0627393 18.0692445  
## 478 34.005714 18.319153 26.2320794 6.8315952  
## 479 38.846566 32.276536 26.1384682 10.7850824  
## 480 7.084622 16.019629 7.8163919 11.4078346  
## 481 17.408520 20.253439 21.3296841 14.2782459  
## 482 39.992468 25.860486 34.5499269 10.4337340  
## 483 40.879539 28.979023 34.5660669 9.2665294  
## 484 10.541278 21.702043 6.0559815 18.4796556  
## 485 31.057692 24.544173 29.2739030 16.3085062  
## 486 38.604229 33.750544 29.7670182 11.6243226  
## 487 34.947690 26.983048 31.2346319 11.7490915  
## 488 30.555136 27.678962 24.8308241 11.7908274  
## 489 40.077643 32.914395 31.0893645 20.3243158  
## 490 38.266556 21.631888 39.9204120 18.2281293  
## 491 38.679420 21.856609 36.3826813 14.8093401  
## 492 37.483018 33.201594 28.1354049 18.5059238  
## 493 52.864396 36.329615 43.1401759 11.8874183  
## 494 31.051891 28.398596 30.7269859 17.3024988  
## 495 5.175125 16.256669 10.7527926 18.0774641  
## 496 46.148147 29.947050 40.7813529 20.2232477  
## 497 41.642230 34.950034 27.1328806 16.0394573  
## 498 44.006982 33.496909 33.4891023 18.5750356  
## 499 22.334840 23.285025 23.8151481 15.6384659  
## 500 50.509809 41.246796 32.9488658 24.0328902  
## 501 16.305322 14.405036 17.0770440 11.5275199  
## 502 34.161435 26.225895 26.2104672 4.8232583  
## 503 28.908221 23.358050 31.7895020 18.9877228  
## 504 45.316125 34.601533 35.7413914 18.8321046  
## 505 31.079227 21.448033 28.4527693 3.8969157  
## 506 40.297484 27.673771 31.8656311 15.7818667  
## 507 37.480379 29.281196 30.5731826 18.6340108  
## 508 34.811503 31.136641 28.7557819 6.1631276  
## 509 36.602219 33.528585 27.0245408 18.4522782  
## 510 47.143025 35.649816 35.3193171 19.2010287  
## 511 47.215486 37.641735 31.8605265 17.8062236  
## 512 35.842969 29.770607 26.3898416 16.6594765  
## 513 38.059006 29.625920 28.7743085 15.0028254  
## 514 34.706095 28.480129 31.1068533 16.6981935  
## 515 56.568133 38.054458 50.5932720 17.2454751  
## 516 29.758322 20.637389 32.2177641 17.0345718  
## 517 12.977204 16.672581 16.6038413 13.7653976  
## 518 53.865376 34.228835 42.3957828 15.5065075  
## 519 40.431897 30.941656 30.1873341 10.5446478  
## 520 2.850499 12.222510 7.8412924 11.0332253  
## 521 42.397670 33.278682 30.5588714 14.1013121  
## 522 39.054145 29.263004 35.5515299 11.8062583  
## 523 10.690836 19.491988 16.3216289 11.9736715  
## 524 41.745527 33.486275 26.7593431 6.5461307  
## 525 41.017238 31.978177 36.2068743 13.9208874  
## 526 39.579288 35.366873 28.7459344 11.6416768  
## 527 36.499404 31.449198 26.1704069 12.7687073  
## 528 13.866738 9.938668 10.6773831 8.2074798  
## 529 39.367788 23.268609 37.3553888 12.8927984  
## 530 37.518307 33.380776 28.1681983 13.6052384  
## 531 51.498073 36.696060 42.2482417 22.0238603  
## 532 34.904640 28.086526 29.6732332 11.9450762  
## 533 34.930390 27.852835 27.0775147 10.5166640  
## 534 16.544134 20.312596 13.0912116 13.1088738  
## 535 27.257659 28.407750 22.1530640 13.9439058  
## 536 17.635281 6.556124 26.1309849 9.0875357  
## 537 19.230006 7.345178 27.5227527 11.9990232  
## 538 36.601380 27.184159 35.5612382 13.0658027  
## 539 36.964769 27.651229 30.4328279 12.9616417  
## 540 3.756262 6.940489 4.7538370 6.1094424  
## 541 38.364427 30.315052 38.8014245 22.3885944

Realizemos una transformación manteniendo sus características

Zscores<-scale(scores)  
transScore <- Zscores\*100+500 # Proceso de baremación de PISA  
transScore <- data.frame(transScore)

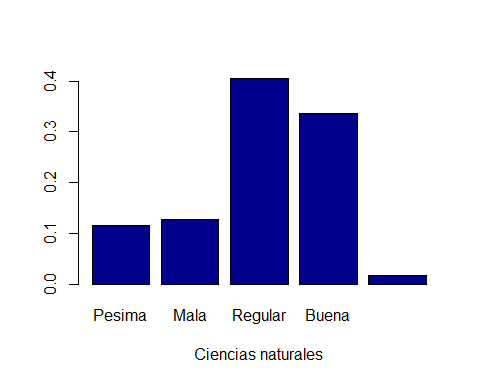
Recodifiquemos para la interpretación

#### RC1 (Ciencias naturales)

transScore$RNC1[transScore$RC1<350] <-1  
transScore$RNC1[transScore$RC1>=350 & transScore$RC1<450] <-2  
transScore$RNC1[transScore$RC1>=450 & transScore$RC1<550] <-3  
transScore$RNC1[transScore$RC1>=550 & transScore$RC1<650] <-4  
transScore$RNC1[transScore$RC1>=650] <-5  
  
  
# Etiquetar  
transScore$RNC1 <- factor(transScore$RNC1,   
 labels = c("Pesima", "Mala", "Regular",  
 "Buena", "Excelente"))  
  
  
fi=table(transScore$RNC1)  
probabilidad=prop.table(table(transScore$RNC1))\*100  
cbind(fi,probabilidad)

## fi probabilidad  
## Pesima 62 11.460259  
## Mala 69 12.754159  
## Regular 219 40.480591  
## Buena 182 33.641405  
## Excelente 9 1.663586

barplot(prop.table(table(transScore$RNC1)), col = "darkBlue", xlab = "Ciencias naturales")



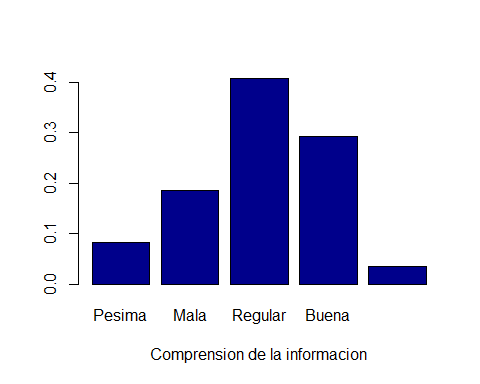
Se observa que la mayor parte se encuentra en un nivel regular y bueno en la parte de ciencias naturales, seguido de malos y pésimos y por último una pequeña parte tiene notas excelentes en las ciencias naturales.

#### RC2 (Comprensión de la información)

transScore$RNC2[transScore$RC2<350] <-1  
transScore$RNC2[transScore$RC2>=350 & transScore$RC2<450] <-2  
transScore$RNC2[transScore$RC2>=450 & transScore$RC2<550] <-3  
transScore$RNC2[transScore$RC2>=550 & transScore$RC2<650] <-4  
transScore$RNC2[transScore$RC2>=650] <-5  
  
  
# Etiquetar  
transScore$RNC2 <- factor(transScore$RNC2,   
 labels = c("Pesima", "Mala", "Regular",  
 "Buena", "Excelente"))  
  
  
fi=table(transScore$RNC2)  
probabilidad=prop.table(table(transScore$RNC2))\*100  
cbind(fi,probabilidad)

## fi probabilidad  
## Pesima 44 8.133087  
## Mala 100 18.484288  
## Regular 220 40.665434  
## Buena 158 29.205176  
## Excelente 19 3.512015

barplot(prop.table(table(transScore$RNC2)), col = "darkBlue", xlab = "Comprension de la informacion")



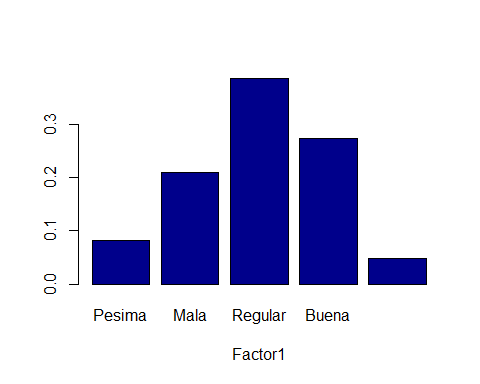
Se observa que la mayor parte se encuentra en un nivel regular, seguido de bueno, mala, pésima y una pequeña parte en el nivel excelente.

#### RC3 (Nivel de actividad fisica)

transScore$RNC3[transScore$RC3<350] <-1  
transScore$RNC3[transScore$RC3>=350 & transScore$RC3<450] <-2  
transScore$RNC3[transScore$RC3>=450 & transScore$RC3<550] <-3  
transScore$RNC3[transScore$RC3>=550 & transScore$RC3<650] <-4  
transScore$RNC3[transScore$RC3>=650] <-5  
  
  
# Etiquetar  
transScore$RNC3 <- factor(transScore$RNC3,   
 labels = c("Pesima", "Mala", "Regular",  
 "Buena", "Excelente"))  
  
  
fi=table(transScore$RNC3)  
probabilidad=prop.table(table(transScore$RNC3))\*100  
cbind(fi,probabilidad)

## fi probabilidad  
## Pesima 44 8.133087  
## Mala 114 21.072089  
## Regular 209 38.632163  
## Buena 148 27.356747  
## Excelente 26 4.805915

barplot(prop.table(table(transScore$RNC3)), col = "darkBlue", xlab = "Factor1")



Se observa que la mayor parte se encuentra en un nivel regular, bueno y malo, más balanceado con respecto a la parte del nivel de actividad física

## CASO 2: Agrupando clientes mayoristas (7 puntos):

El conjunto de datos se refiere a los clientes de un distribuidor mayorista de Portugal, el cual comercializa distintos tipos de productos.

Cada una de las observaciones hace referencia a un cliente distinto, el cual incluye el gasto anual en unidades monetarias (u.m.) para cada una de las categorías.

Se nos solicita realizar un análisis clúster que nos permita agrupar a nuestros clientes en función de los distintos tipos de productos que adquirieron, para lo cual contamos:

| Variable | Descripción |
| --- | --- |
| Channel | Canal de clientes: 1. Horeca (Hotel/Restaurante/Café) 2. Canal Minorista |
| Región | Región de los clientes: 1. Lisboa, 2. Oporto y 3. Otra |
| Fresh | Gasto anual en productos frescos. |
| Milk | Gasto anual en productos lácteos. |
| Grocery | Gasto anual en productos comestibles. |
| Frozen | Gasto anual en productos congelados. |
| Detergent\_Papers | Gasto anual en detergentes y productos de papel. |
| Delicatessen | Gasto anual en productos preparados (snacks y licor). |

Los datos se encuentran en el archivo “clientes.csv”.

clientes <- read.csv(here("9 Analisis Multivariado/Trabajo 3/clientes.csv"))

Luego de cargar el conjunto de datos en R, realizar las 2 opciones que se presenta:

**Opción 1**:

1. Generar un nuevo dataset solo con las variables numéricas y estandarizarlas.
2. Generar el agrupamiento por particiones utilizando el método kmeans con k=4.
3. Añadir el dataset original la columna cluster, que identificará a los grupos que obtuvimos mediante esta metodología.
4. Graficar y perfilar a nuestros clientes según su agrupación.

**Opción 2**:

1. Generar un nuevo dataset solo con las variables numéricas y estandarizarlas.
2. Encuentre ahora los clusters de forma jerárquica, calculando la matriz de distancias euclidianas y seleccionando en enlace que creas mejor se ajuste a los datos.
3. Comparar los métodos de enlace y determinar cuál es el adecuado.
4. Generar el nuevo agrupamiento jerárquico con el enlace seleccionado.
5. Graficar el dendograma respectivo y determinar el número de clusters.
6. Graficar y perfilar a nuestros clientes según su agrupación jerárquica.

### Solución

**Opción 1**:

1. **Generar un nuevo dataset solo con las variables numéricas y estandarizarlas.**
2. **Generar el agrupamiento por particiones utilizando el método kmeans con k=4.**
3. **Añadir el dataset original la columna cluster, que identificará a los grupos que obtuvimos mediante esta metodología.**
4. **Graficar y perfilar a nuestros clientes según su agrupación.**

**Opción 2**:

1. **Generar un nuevo dataset solo con las variables numéricas y estandarizarlas.**
2. **Encuentre ahora los clusters de forma jerárquica, calculando la matriz de distancias euclidianas y seleccionando en enlace que creas mejor se ajuste a los datos.**
3. **Comparar los métodos de enlace y determinar cuál es el adecuado.**
4. **Generar el nuevo agrupamiento jerárquico con el enlace seleccionado.**
5. **Graficar el dendograma respectivo y determinar el número de clusters.**
6. **Graficar y perfilar a nuestros clientes según su agrupación jerárquica.**

## CASO 3: (7 puntos)

Investigar y realizar un informe monográfico sobre el ***análisis de correspondencia múltiple*** adjuntar un ejercicio aplicando R o Phyton.

### Solución

Aca va la solución